

Approximating Probability Distributions Done Right

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Disclaimer: This post is written based on the syllabus for Cambridge International AS and A Levels, and may not be applicable for higher level studies.

Approximation

1. Binomial to Normal:

- Conditions*: $np > 5$ and $nq > 5$ (n is sufficiently large).
- Apply continuity correction.

2. Binomial to Poisson:

- A Poisson distribution can be used to model a discrete probability distribution in which the events
 - occur singly,
 - at random and independently,
 - in a given interval of space or time.
- The mean and variance of a Poisson distribution are equal.
- Conditions*: $n > 50$ and $np < 5$ (n is large, p is small/rare event).

3. Poisson to Normal:

- Conditions*: $\lambda > 15$
- Apply continuity correction.

Central Limit Theorem

The central limit theorem (CLT) states that, provided n is large, the distribution of **sample means** of size n is:

$$\bar{X}(n) \sim N\left(\mu, \frac{\sigma^2}{n}\right),$$

where the original population has mean μ and variance σ^2 .

- CLT can be used for sample size $n > 50^*$ (or $n > 30^{**}$).

*Rule of thumb

**For Further Statistics

References:

1. Chalmers, Dean. Cambridge International AS & A Level Mathematics: Probability & Statistics 1 - Coursebook. Cambridge University Press, 2018.
2. Kranat, Jayne. Cambridge International AS & A Level Mathematics: Probability & Statistics 2 - Coursebook. Cambridge University Press, 2018.
3. McKevley, Lee, and Crozier, Martin. Cambridge International AS & A Level Further Mathematics - Coursebook. Cambridge University Press, 2018.