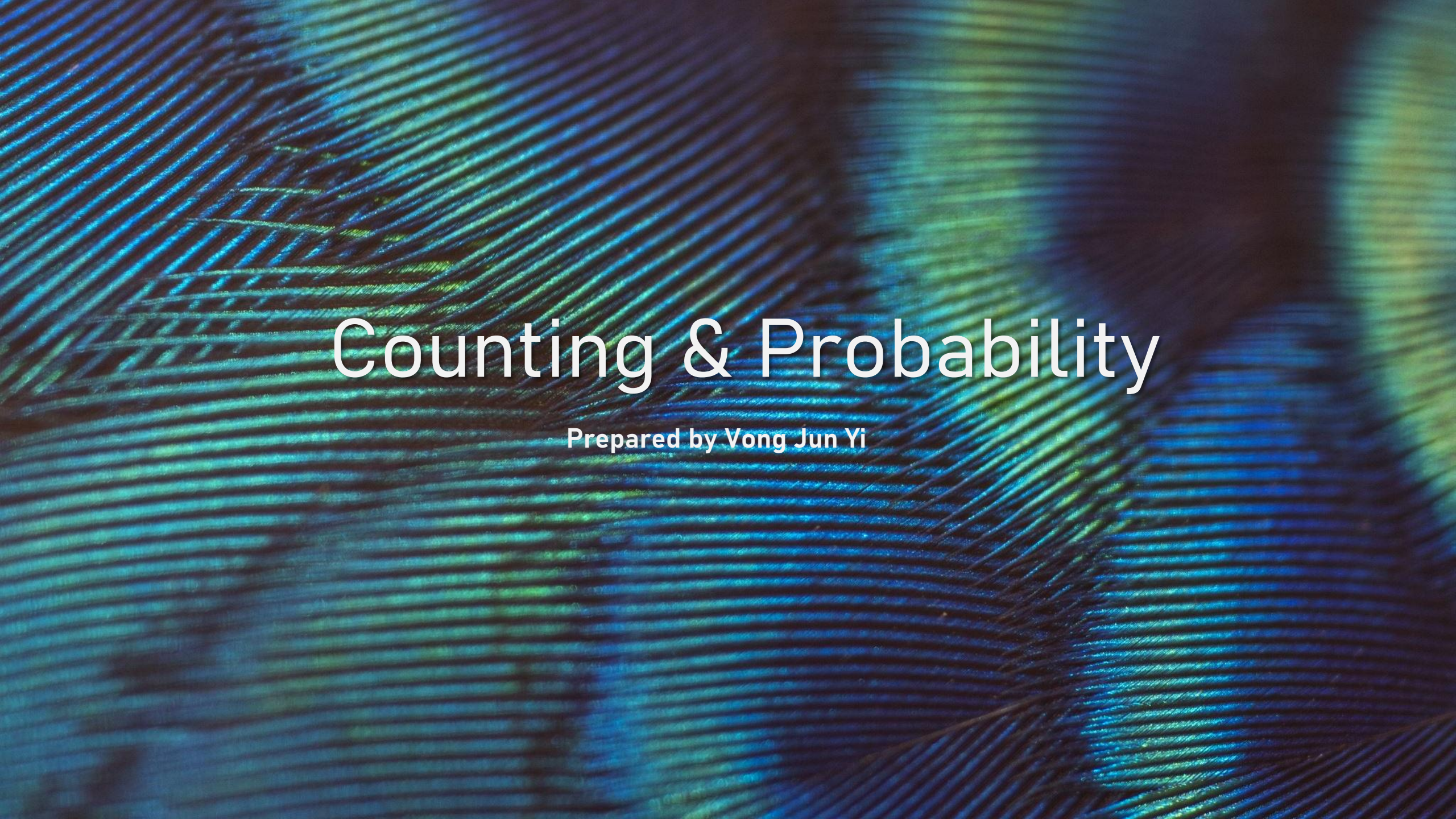
Welcome!
The meeting will start shortly...

Try out this problem first! (Solution will be revealed at the end of the meeting)

5. A Badminton Club organises a competition among its members. A total of 38 members are involved in this competition. 20 members play in the double event and 26 members play in the single event. Calculate the number of members who play in both events.



Counting & Probability

Prepared by Vong Jun Yi

What are sets?

In a vague sense, a set is...

- A collection of different things, known as **elements**, which can be objects of any kind (e.g. numbers, symbols, points in space, etc.)

A set containing different kinds of polygons

Let's recall...

If we are given a set A,

- **Universal set**, ξ – A set which contains all objects, including itself.
- **Complement** of set A, A' – The set of all elements which is not contained within A.
- **Cardinality** of A, |A| – The number of elements in A.
- $B \subset A$ means B is a **subset** of A ($B \not\subset A$ means B is not a subset of A)
- $x \in A$ means x is an **element** of A ($x \notin A$ means x is not an element of A)

Warm-up problems

Solve the problems below under 4 minutes.

1. Let set M = {1,2,3}. How many possible subsets of M are there?
2. Given $\xi = \{x \mid 53 \leq x \leq 62, x \text{ is an integer}\}$, P is the set of integers such that the sum of both digits is a prime number. Find the number of elements of set P'.

Solution to warm-up problems

1. Let set M = {1,2,3}. How many possible subsets of M are there?

- First approach: Start by listing out all possible subsets.

- Second approach: In general, the number of possible subsets for any set is 2^n, where n is the cardinality of the set.

Solution to warm-up problems

2. Given $\xi = \{x \mid 53 \leq x \leq 62, x \text{ is an integer}\}$, P is the set of integers such that the sum of both digits is a prime number. Find the number of elements of set P'.

{53, 54, 55, 56, 57, 58, 59, 60, 61, 62}

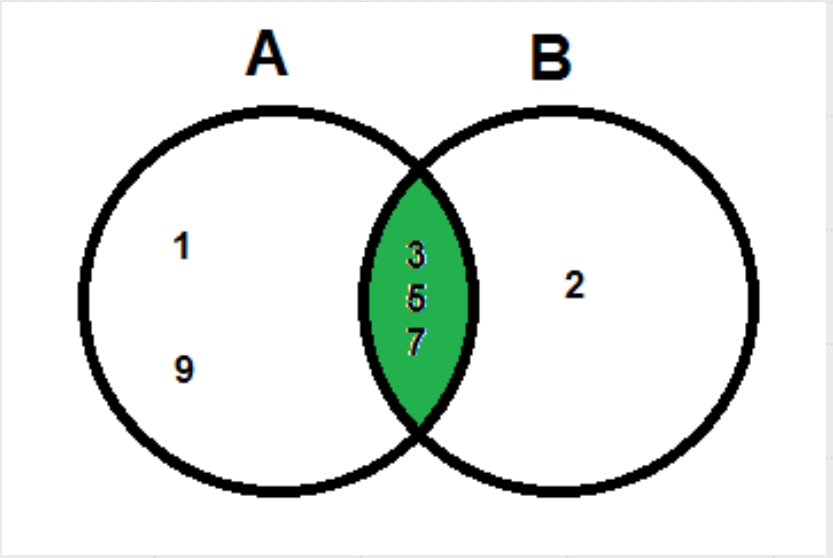
Sum of digits

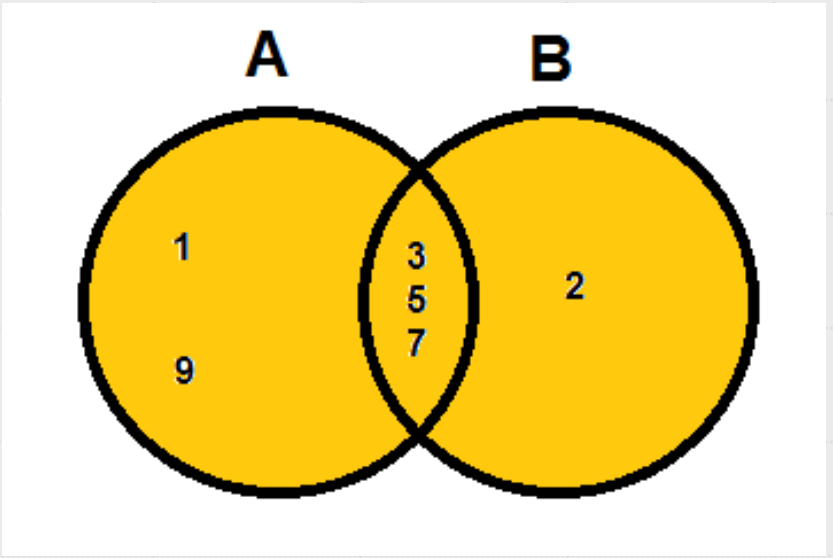
{8, 9, 10, 11, 12, 13, 14, 6, 7, 8}

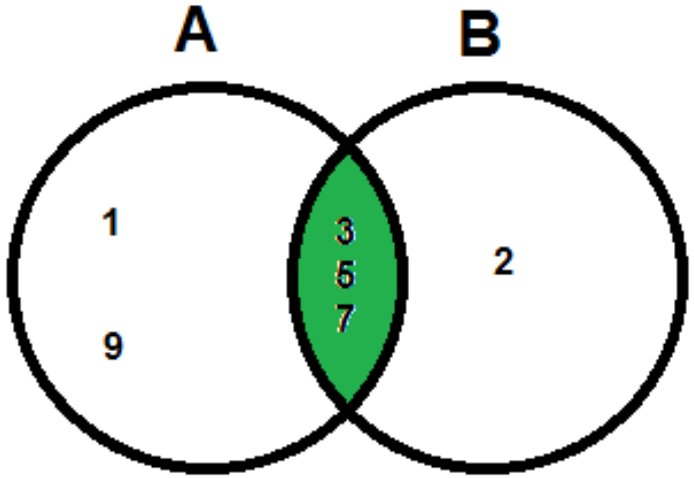
There are 3 integers in P,
so the cardinality of P' is **(62-53+1)-3 = 7**

What are unions and intersections?

Refer to the Venn diagrams on the right.
Can you tell which of them is an **intersection**?





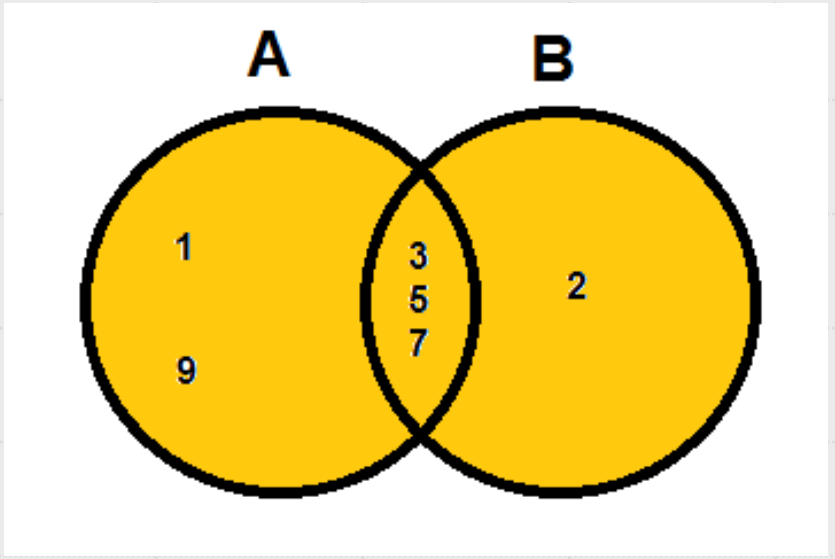


Intersection of A and B

$A \cap B = \{3,5,7\}$

Union of A and B

$A \cup B = \{1,2,3,5,7,9\}$



Principle of Inclusion-Exclusion (PIE)

- **Lemma 1** Given sets A and B, the cardinality of the union of A and B is

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Problems #1

- **Lemma 1** Given sets A and B, the cardinality of the union of A and B is

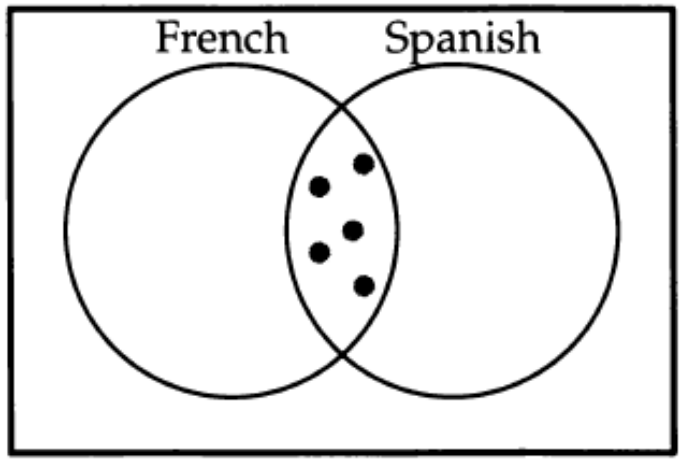
$$|A \cup B| = |A| + |B| - |A \cap B|$$

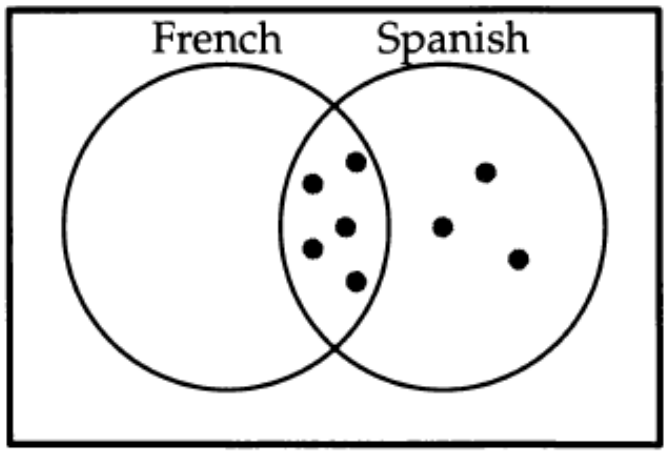
Problem 1.7: At Brown High School, there are 12 players on the basketball team. All of the players are taking at least one foreign language class. The school offers only Spanish and French as its foreign language classes. 8 of the players are taking Spanish and 5 of the players are taking both languages. How many players are taking French?

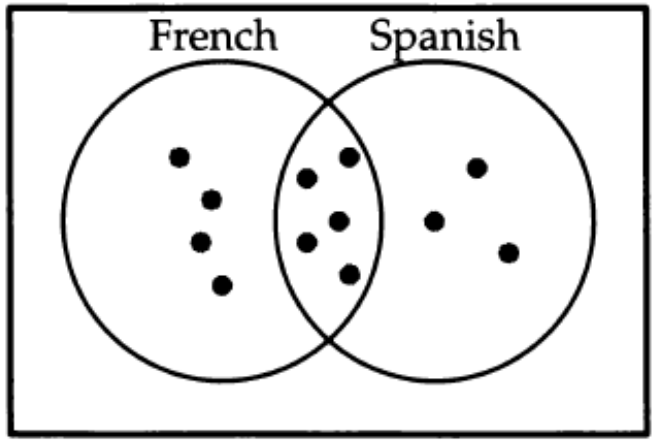
Problem 1.8: There are 27 cats at the pound. 14 of them are short-haired. 11 of them are kittens. 5 of them are long-haired adult cats (not kittens). How many of them are short-haired kittens?

Solution to Problem 1.7

Problem 1.7: At Brown High School, there are 12 players on the basketball team. All of the players are taking at least one foreign language class. The school offers only Spanish and French as its foreign language classes. 8 of the players are taking Spanish and 5 of the players are taking both languages. How many players are taking French?







Shortcut?

Problem 1.7: At Brown High School, there are 12 players on the basketball team. All of the players are taking at least one foreign language class. The school offers only Spanish and French as its foreign language classes. 8 of the players are taking Spanish and 5 of the players are taking both languages. How many players are taking French?

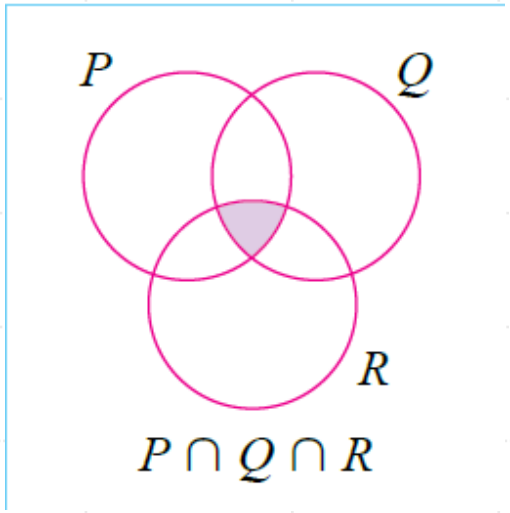
$|A \cup B| = 12, |A| = 8, |A \cap B| = 5, |B| = ?$

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$12 = 8 + |B| - 5$$
$$\therefore |B| = 9$$

Principle of Inclusion-Exclusion (PIE)

- **Lemma 2** Given sets A, B, and C, the cardinality of the union of A, B and C is

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$



Problems #2

A total of 100 adults are involved in a survey on their top choices of reading materials. 40 people choose newspapers, 25 people choose magazines, 18 people choose storybooks, 8 people choose both newspapers and magazines, 7 people choose both magazines and storybooks, 5 people choose both newspapers and storybooks, and 3 people choose all three types of reading materials. How many people do not choose any of the reading materials?

The Form 5 pupils who are involved in Recycling Programme manage to collect old newspapers, plastic bottles and tins. 72 pupils collect plastic bottles, 36 pupils collect old newspapers, 25 pupils collect tins, 20 pupils collect old newspapers and plastic bottles, 8 pupils collect old newspapers and tins, 18 pupils collect plastic bottles and tins, and 7 pupils collect all the three types of materials. Calculate the total number of pupils are involved in the programme.

Principle of Inclusion-Exclusion (PIE)

- Can you find a generalization for PIE?
- **Theorem** The principle of inclusion-exclusion states that for finite sets $A_1, A_2, \ldots, A_n$, one has the identity

$$\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{i=1}^{n} |A_i| - \sum_{1 \leqslant i < j \leqslant n} |A_i \cap A_j| + \sum_{1 \leqslant i < j < k \leqslant n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap \cdots \cap A_n|.$$

Final quiz

Please write your answers on a sheet of paper.
Working is optional, no extra points will be awarded for working.
You will be required to submit it to the teacher advisors for grading once complete.

1.3.1 There are 20 cars in my building's parking lot. All of the cars are red or white. 12 of them are red, 15 of them are 4-door, and 4 of them are 2-door and white. How many of the cars are 4-door and red?

1.3.2 Going back to the 12-person basketball team from Problem 1.7, all 12 players are taking at least one of biology or chemistry. If 7 players are taking biology and 2 players are taking both sciences, how many players are taking chemistry?

1.3.3 There are 30 students in Mrs. Taylor's kindergarten class. If there are twice as many students with blond hair as with blue eyes, 6 students with blond hair and blue eyes, and 3 students with neither blond hair nor blue eyes, how many students have blue eyes?

1.3.4⋆ At the Gooddog Obedience School, dogs can learn to do three tricks: sit, stay, and roll over. Of the dogs at the school:

50 dogs can sit	17 dogs can sit and stay
29 dogs can stay	12 dogs can stay and roll over
34 dogs can roll over	18 dogs can sit and roll over
9 dogs can do all three	9 dogs can do none

(a) How many dogs are in the school?

(b) How many dogs can do exactly 2 tricks?

1.3.5⋆ Every student in my school is in either French class or Spanish class, or both. Let x be the number of students in French class, y be the number of students in Spanish class, and z be the number of students that are in both classes. Find an expression in terms of x, y, and z for how many students there are in my school.

Thank you

Try this **evil** question!

A school has four clubs, A, B, C, D, whose member are students in this school. Every two clubs have 227 common members. Every three clubs have 117 common members. There are exactly 17 students that join all four clubs. At least how many students does club A have?